1. A spam filter is designed by looking at commonly occurring phrases in spam. Suppose that 80% of email is spam. In 10% of the spam emails, the phrase “free money” is used, whereas this phrase is only used in 1% of non-spam emails. A new email has just arrived, which does mention “free money”. What is the probability that it is spam?

Alice attends a small college in which each class meets only once a week. She is deciding between 30 non-overlapping classes. There are 6 classes to choose from for each day of the week, Monday through Friday. Trusting in the benevolence of randomness, Alice decides to register for 7 randomly selected classes out of the 30, with all choices equally likely. What is the probability that she will have classes every day, Monday through Friday? (This problem can be done either directly using the naive definition of probability, or using inclusion-exclusion.)

Direct Method: There are two general ways that Alice can have class every day: either she has 2 days with 2 classes and 3 days with 1 class, or she has 1 1 day with 3 classes, and has 1 class on each of the other 4 days. The number of possibilities for the former is 5 2 6 2 2 63 (choose the 2 days when she has 2 classes, and then select 2 classes on those days and 1 class for the other days). The number of possibilities for the latter is 5 1 6 3 64. So the probability is 5 2 6 2 2 63 + 5 1 6 3 64 30 7 = 114 377 ⇡ 0.302. Inclusion-Exclusion Method: we will use inclusion-exclusion to find the probability of the complement, which is the event that she has at least one day with no classes. Let Bi = Ac i. Then P(B1 [ B2 ··· [ B5) = X i P(Bi) X i<j